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RADIANT HEAT TRANSFER IN A CLOSED SYSTEM OF SEMIOPAQUE BODIES SEPARATED BY AN EMITTING AND ABSORBING GAS MEDIUM

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UDC 536.3

A radiant heat-transfer problem is solved for a closed emitting system bounded by a nonisothermal semiopaque shell with the absorption and emission of a nonisothermal gas medium taken into account.

Analysis [1-4] shows that the solution of radiant heat-transfer problems at this time is performed for systems of bodies opaque to thermal radiation.

The extensive utilization of films and plastics in the construction of modern structures evokes the necessity to solve radiant heat-transfer problems for systems of semiopaque bodies relative to thermal radiation. Moreover, the space of these structures is filled with a nonisothermal medium emitting and absorbing thermal radiation since triatomic gases are usually contained therein. The cause of the nonisothermy of the gas space is the different temperature of its bounding surfaces.

We solve the problem formulated in general form first for boundary conditions of the first kind. As is known, in this case the temperatures of the body surfaces and the gas medium are given in this case. It is required to determine the resultant radiation Q_r of each of the elements of the emitting system.

Let us consider a nonisothermal semiopaque shell in which a nonisothermal medium is enclosed. In order to make an assumption about the diffuse nature of the emission, we divide the shell and the gas, respectively, into n isothermal surfaces and m = n + 1 isothermal spaces, as is shown in Fig. 1. The surfaces and the gas spaces are assumed gray.

Each of the n surfaces of the closed system has the temperature T_i and the following integral hemispherical radiation characteristics: ε_i, A_i, D_i. The temperature of the i-th space of the medium equals T_{gi} and its integral hemispherical radiation characteristics for the temperature T_{gj} and T_j have the respective values $\varepsilon_{gi,j}$, $a_{gi,j}$, $d_{gi,j}$ and $\varepsilon_{gi,j}$, $A_{gi,j}$, $D_{gi,j}$. Because there are no suspended particles in the gas medium we consider the energy scattering effect not to hold and, therefore, $d_{gi,j} = 1 - a_{gi,j}$; and $D_{gi,j} = 1 - A_{gi,j}$.

In connection with the fact that the surfaces and gas spaces are diffuse, we characterize the geometry of the body system by the mean angular coefficients φ_{j-i}, φ_{gj-i}, φ_{i-gj}, φ_{gi-gj}. The generalized angular coefficients are here determined by using the expression $\psi = D\varphi$, since the transmissivity D is taken out from under the integral.

According to Fig. 2, the resultant emission for each of the n semiopaque surfaces can be represented in the form

$$Q_{r,i} = Q_{ie,i} - Q_{ef,i} - Q_{t,i} = p_i Q_{ie,i} - Q_{ef,i}. \tag{1}$$

All-Union Central Scientific-Research and Design Institute "Gipronisel'prom", Orel.
Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 54, No. 2, pp. 305-309, February, 1988. Original article submitted October 16, 1986.

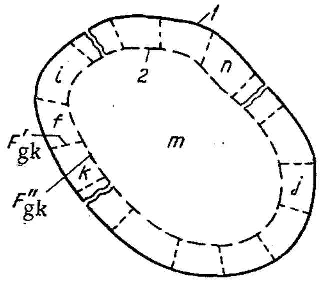

Fig. 1

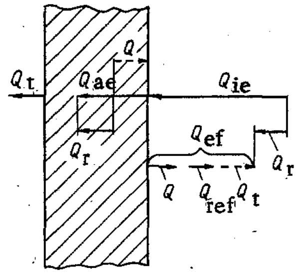

Fig. 2

Fig. 1. Closed system of n isothermal semiopaque bodies and m isothermal gas spaces: 1) semiopaque shell surface; 2) gas space surface; m = n + 1; k = i − 2; f = k + 1 = i − 1.

Fig. 2. Diagram of radiant fluxes (W) during heat transfer of a semiopaque body to the environment.

In (1)

$$Q_{t,i} = D_i Q_{ie,i};\ p_i = 1 - D_i;\ Q_{ef,i} = \sum_{j=1}^{n} Q_{ef,i}\varphi_{i-j};$$

$$Q_{ie,i} = \sum_{j=1}^{n} D_{j-i} g_{ef,j} F_j \varphi_{j-i} + \sum_{j=1}^{m} D_{gj-i}\varepsilon_{gj,j} E_{0,gj} F^{*}_{gj}\varphi_{gj-i}, \quad (2)$$

where D_{j-i}, D_{gj-i} are the reduced transmissivities of the nonisothermal gas medium relative to the radiant fluxes, respectively, from the j-th surface and from the j-th gas space on the surface i, and F_{gj}^{*} is the area of the surface of the gas space j visible from the surface i, and $E_{0,gj} = \sigma_0 T_{gj}^4$.

The values of the quantities D_{j-i} and D_{gj-i} naturally depend on the arrangement of the appropriate pair of surfaces in the body system in each specific case and on the whole on the geometry of the system itself. As an illustration we consider certain cases of determining these quantities (see Fig. 1)

$$D_{j-i} = D_{gj,j} D_{gm,j} D_{gi,j};\ D_{gj-i} = d_{gm,j} d_{gi,j};$$
$$D_{i-i} = D_{gi,i};\ D_{gi-i} = 1,\ F^{*}_{gi}\varphi_{gi-i} = F_i;$$
$$D_{k-i} = D_{gk,k} D_{gf,k} D_{gi,k};\ D_{gk-i} = d_{gv,k} d_{gi,k},\ d_{gv,k} = (d_{gf,k} F'_{gk}\varphi'_{gk-i} + d_{gm,k} F''_{gk}\varphi''_{gk-i})/(F'_{gk} + F''_{gk})\,\varphi_{gk-i}, \quad (3)$$

where, say, $D_{gm,j}$ and $d_{gm,j}$ are the transmissivities of parts of the gas space m located on the path between the surfaces j and i, respectively, at the temperatures T_j and T_{gj}; F_{gk}' and F_{gk}'' are shown in Fig. 1.

In order to simplify the solution of the problem, we eliminate the quantities $Q_{ef,i}$ and $Q_{ef,j}$ in (1). To do this, we set up a relationship between the effective emission of a semiopaque body and its resultant emission analogous to that obtained in [4] for opaque bodies. Taking into account that $Q_{ie,i} = Q_{ae,i}/A_i$ and $Q_{ae,i} = Q_{r,i} + Q_i$ (see Fig. 2), we obtain from (1)

$$Q_{ef,i} = p_{*i} E_{0,i} F_i + R_{*i} Q_{r,i}, \quad (4)$$

where $p_{*i} = p_i\varepsilon_i/A_i;\ R_{*i} = (1/A_i - D_i/A_i - 1);\ E_{0,i} = \sigma_0 T_i^4$.

Taking account of (2) and (4), after appropriate manipulation Eq. (1) finally takes the following form:

$$\sum_{j=1}^{n} a_{ij} Q_{r,j} = b_i \quad (i = 1, 2, \ldots, n), \quad (5)$$

where

$$a_{ii} = 1 + R_{*i}\left[\sum_{\substack{j=1 \\ j\neq i}}^{n} \varphi_{i-j} + (1 - p_i D_{gi,i})\,\varphi_{i-i}\right];$$

$$a_{ij} = -p_i D_{j-i} R_{*j}\varphi_{j-i};\ b_i = \sum_{\substack{j=1 \\ j\neq i}}^{n} (p_i p_{*j} D_{j-i} E_{0,j} - p_{*i} E_{0,i})\, F_i \varphi_{i-j} +$$

$$+ p_{*i}(p_i D_{gi,i} - 1)\, E_{0,i} F_i \varphi_{i-i} + \sum_{j=1}^{m} p_i D_{gj-i} \varepsilon_{gj,j} E_{0,gj} F_i \varphi_{i-gj}.$$

The subscript i in the system (5) indicates the number of the equation and the subscript j is the number of the unknown quantity Q_r.

In matrix form the system (5) is

$$A\mathbf{Q} = \mathbf{b}, \text{ from which } \mathbf{Q} = A^{-1}\mathbf{b}, \tag{6}$$

where $A = [a_{ij}]$ is the initial square matrix, A^{-1} is the inverse matrix, Q is a column vector whose elements are the unknown members $Q_{r,j}$, and **b** is a column vector whose elements are the free terms b_i.

The system of linear equations (5) can be solved by different methods [5] by using standard programs for electronic computer computations. For $i \leqslant 3$ the Kramer formula can be used [5].

To determine the resultant emission of the j-th space of the gas medium we use (4) as well as the known relationships

$$Q_{r,gj} = Q_{ie,gj} - Q_{ef,gj};\ Q_{ie,gj} = \sum_{i=1}^{n} D_{i-gj} Q_{ef,i} \varphi_{i-gj} + \sum_{i=1}^{m} D_{gi-gj} \varepsilon_{gi,i} E_{0,gi} F^*_{gi} \varphi_{gi-gj};\ (4);\ Q_{ef,gj} = Q_{gj} + Q_{t,gj} =$$

$$= \sum_{i=1}^{m} \varepsilon_{gj,j} E_{0,gj} F^*_{gj} \varphi_{gj-gi} + \sum_{i=1}^{n} D_{gj,i} D_{i-gj} Q_{ef,i} \varphi_{i-gj} + \sum_{i=1}^{m} d_{gj,i} D_{gi-gj} \varepsilon_{gi,i} E_{0,gi} F^*_{gi} \varphi_{gi-gj}.$$

Then after appropriate manipulations we obtain

$$Q_{r,gj} = \sum_{i=1}^{n} A_{gj,i} D_{i-gj} (R_{*i} Q_{r,i} + p_{*i} E_{0,i} F_i)\, \varphi_{i-gj} + \sum_{i=1}^{m} (a_{gj,i} D_{gi-gj} \varepsilon_{gi,i} E_{0,gi} - \varepsilon_{gj,j} E_{0,gj})\, F^*_{gj} \varphi_{gj-gi}, \tag{7}$$

where D_{i-gj} and D_{gi-gj} are the reduced transmissivities of the nonisothermal gas medium relative to the radiant fluxes, respectively, from the i-th surface and from the i-th gas space in the gas space j, $Q_{r,gj} = g_{r,gj} F_{gj}$; F_{gj}, F_{gj}^* is the area of the surface of the gas space j, respectively, of the total and the visible gas space i from the surface.

The values of the quantities D_{i-gj} and D_{gi-gj} are determined by a method analogous to the determination presented earlier for the quantities D_{j-i} and D_{gj-i}.

The emission Q_t transmitted by semiopaque bodies is determined from the heat-balance equation

$$\sum_{j=1}^{n} Q_{r,j} + \sum_{j=1}^{m} Q_{r,gj} + Q_t = 0. \tag{8}$$

If the gas medium is isothermal and its radiation characteristics at the temperatures T_g and T_i have the respective values ε_g, a_g, d_g and $\varepsilon_{g,i}, A_{g,i}, D_{g,i}$, then the coefficients in the system (5) take the form

$$a_{ii} = 1 + R_{*i}\left[\sum_{\substack{j=1\\ j\neq i}}^{n} \varphi_{i-j} + (1 - p_i D_{g,i})\varphi_{i-i}\right];$$

$$a_{ij} = -p_i D_{g,j} R_{*j} \varphi_{j-i};$$

$$b_i = \sum_{\substack{j=1\\ j\neq i}}^{n} (p_i p_{*j} D_{g,j} E_{0,j} - p_{*i} E_{0,i}) F_i \varphi_{i-j} + p_{*i}(p_i D_{g,i} - 1) E_{0,i} F_i \varphi_{i-i} + p_i \varepsilon_g E_{0,g} F_i. \quad (9)$$

The resultant emission by the isothermal gas medium is determined as follows:

$$Q_{r,g} = \sum_{i=1}^{n} A_{g,i} R_{*i} Q_{r,i} + \sum_{i=1}^{n} (p_{*i} A_{g,i} E_{0,i} - \varepsilon_g E_{0,g}) F_i, \quad (10)$$

where $Q_{r,g} = g_{r,g} \sum_{i=1}^{n} F_i$.

For gray surfaces and a gray gas according to Kirchhoff's law, $A_i = \varepsilon_i$ and $A_{g,i} = \varepsilon_{g,i} (a_g = \varepsilon_g)$, while $p_{*i} = p_i$.

NOTATION

T, absolute temperature, K; ε, emissivity; A, a, absorptivity, respectively, at the body surface and gas medium temperatures; D, d, transmissivity, respectively, at the body surface and gas medium temperatures; Q, radiant flux, the inherent body emission, W; g, emission flux density, W/m²; E_0, density of the hemispherical integral emission of an absolutely black body, W/m²; φ, irradiation factor from one surface onto another; F, surface area participating in radiation heat transfer, m²; σ_0, Stefan–Boltzmann constant, $W/(m^2 \cdot K^4)$. Subscripts: 1, 2, ..., n, ordinal numbers of the body (surface) participating in the radiation heat transfer; gl, g2, ..., gn, gm, rodinal number of the gas space, m = n + 1; g and g,i, gas medium, respectively, at the temperatures T_g and T_i; gi,j, gas space i at the temperature T_{gj} (or T_j); r, resultant surface emission; ie, emission incident on the surface; ef, effective surface emission; t, emission transmitted by the body; ae, emission absorbed by the body, ref, emission reflected by the surface; sh, shell.

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